

Dynamic Influence Analysis in Evolving Networks

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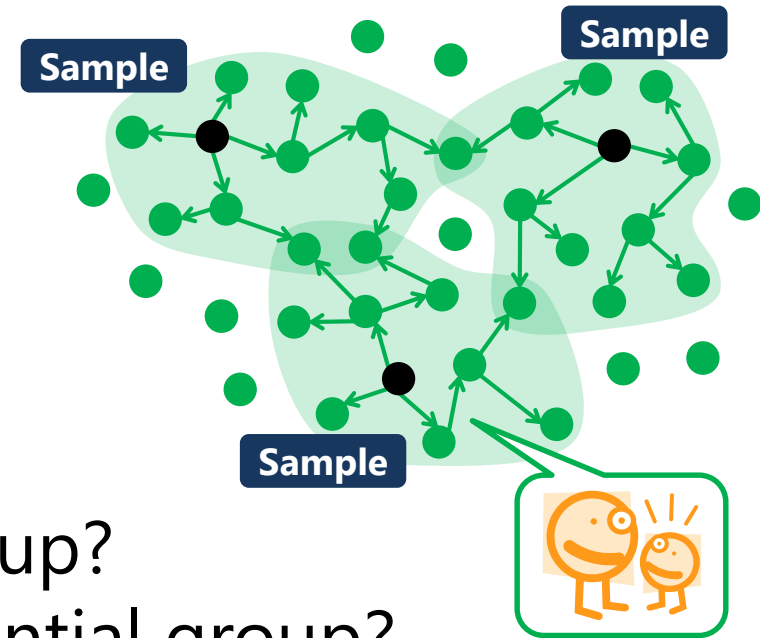
Ken-ichi Kawarabayashi (NII)

Influence analysis in online social networks

Application: viral marketing

[Domingos-Richardson. *KDD'01*]

Product promotion through word-of-mouth effects



Q. How influential is a given group?

Q. How to select the most influential group?

Graph problems

Influence estimation
Influence maximization

[Kempe-Kleinberg-Tardos. *KDD'03*]

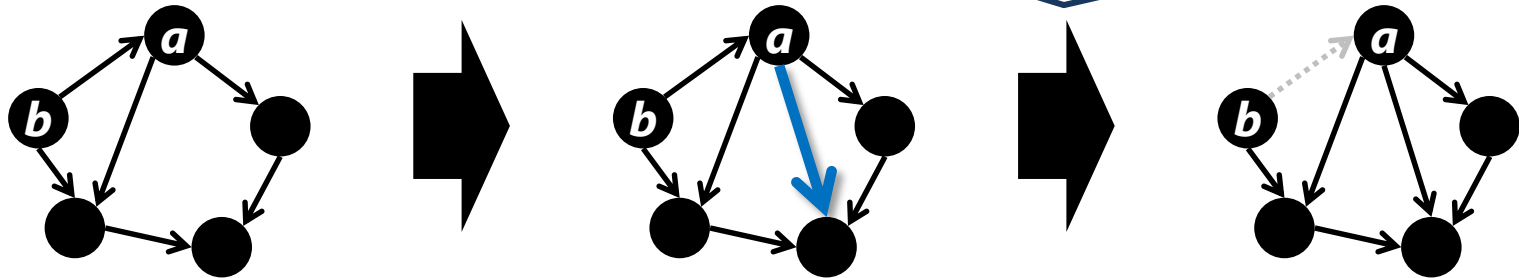
Existing algorithms for influence maximization (2003—2015)

Strategy	Scalability	Accuracy
Simulation [Kempe-Kleinberg-Tardos. <i>KDD'03</i>] [Kimura-Saito-Nakano. <i>AAAI'07</i>] [Chen-Wang-Yang. <i>KDD'09</i>] ✓our previous work [O.-Akiba-Yoshida-Kawarabayashi. <i>AAAI'14</i>]	< 100M edges Quadratic time	Good ≈ 63% approx.
Heuristics [Chen-Wang-Yang. <i>KDD'09</i>] [Chen-Wang-Wang. <i>KDD'10</i>] [Jung-Heo-Chen. <i>ICDM'12</i>]	100M – 1B edges	Bad No guarant.
Sketching [Borgs-Brautbar-Chayes-Lucier. <i>SODA'14</i>] [Tang-Xiao-Shi. <i>SIGMOD'14</i>] [Tang-Shi-Xiao. <i>SIGMOD'15</i>]	> 1B edges Near-linear time	Good ≈ 63% approx.

These algorithms are **static**
 Real-world social networks are **dynamic**

Social networks are **dynamic** and **evolving**

Accounts and friendships appear or disappear



Want to **track** influential vertices

Simply applying static methods \rightsquigarrow **> Linear time**

☹️ Methods are almost **undeveloped**

[Zhuang-Sun-Tang-Zhang-Sun. *ICDM'13*]

Probing a small number of vertices

[Chen-Song-He-Xie. *SDM'15*] Edge operations only

Our contribution

Fully-dynamic indices

for influence analysis in evolving networks

① Indexing

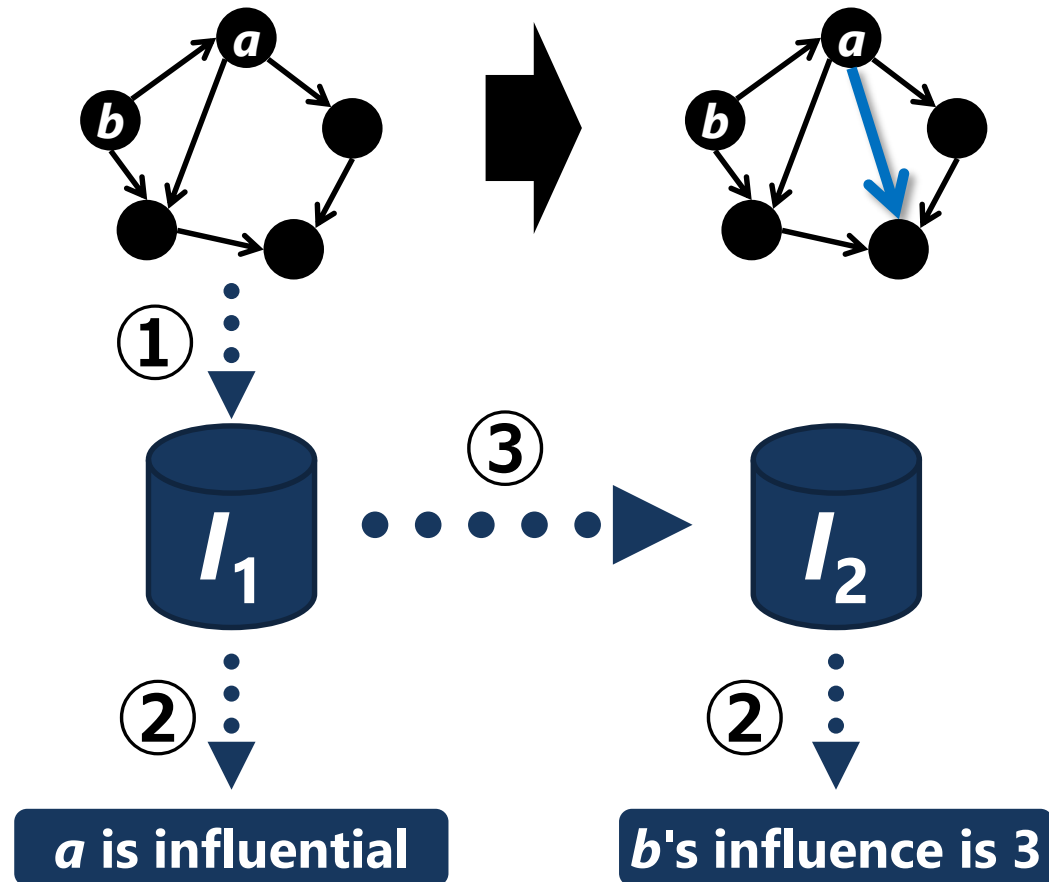
Almost-linear size

② Analysis query

Accuracy guarant.

③ Index update

Any change



Problem definition

Diffusion model: Independent cascade

[Goldenberg-Libai-Muller. *Market. Lett.* '01]

Graph $G = (V, E, p)$ with edge probabilities

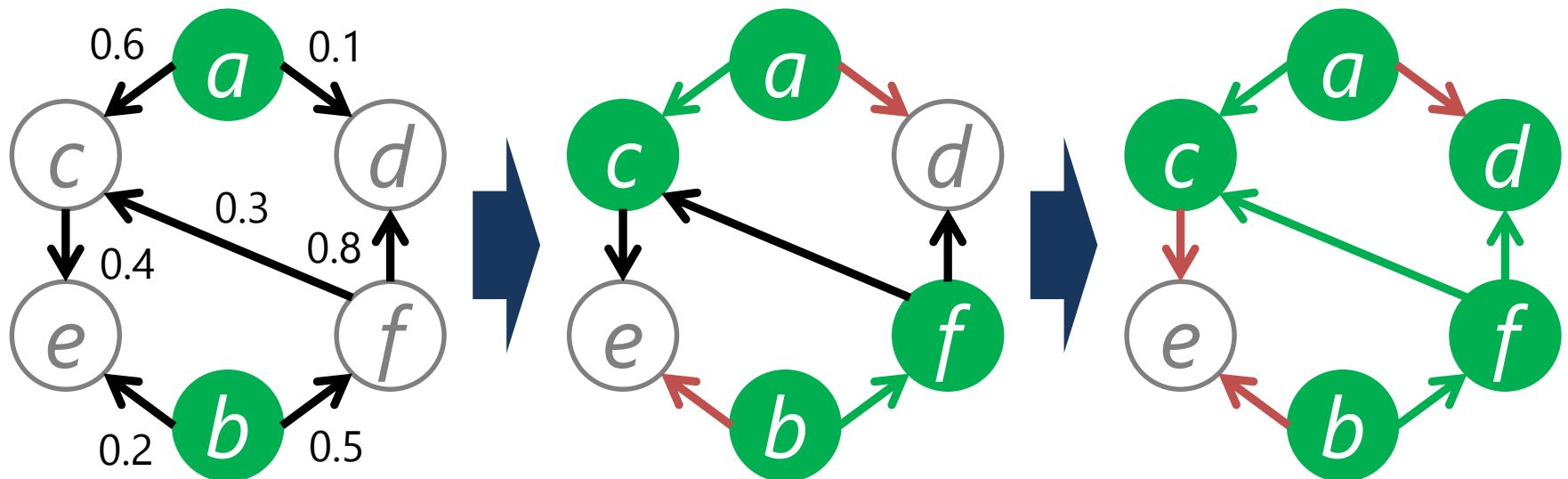
Seed set $S \subseteq V$

Initialize vertex's state

- ▶ **Active** if $\in S$
- ▶ **Inactive** if $\notin S$

Active u to **Inactive** v

- ▶ **Success** w.p. p_{uv}
- ▶ **Failure** w.p. $1 - p_{uv}$



Problem definition

Diffusion model: Independent cascade

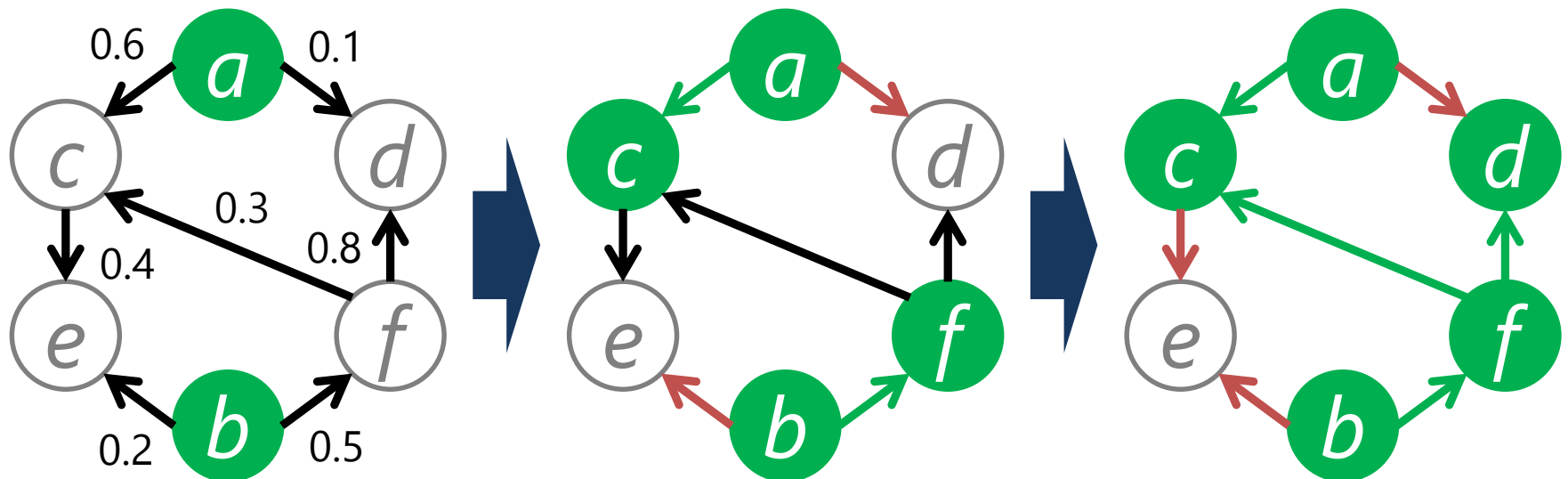
[Goldenberg-Libai-Muller. *Market. Lett.* '01]

Graph $G = (V, E, p)$ with edge probabilities

Seed set $S \subseteq V$

Influence spread

$$\sigma(S) := \mathbf{E}[\# \text{ active vertices given } S]$$



Problem definition

Influence estimation

Input seed set S

Output $\sigma(S)$

#P-hard

[Chen-Wang-Wang. *KDD'10*]

Monte-Carlo is **good approx.**

Influence maximization

[Kempe-Kleinberg-Tardos. *KDD'03*]

Input integer k
 $\operatorname{argmax} \sigma(S)$

Output $S: |S|=k$

NP-hard [Kempe+'03]

Greedy strategy is

$(1 - e^{-1}) \approx$ **63%-approx.**

[Nemhauser-Wolsey-Fisher. *Math. Program.*'78]

$\sigma(\cdot)$ is **submodular** [Kempe+'03]

$$\forall X \subseteq Y, v \notin Y$$

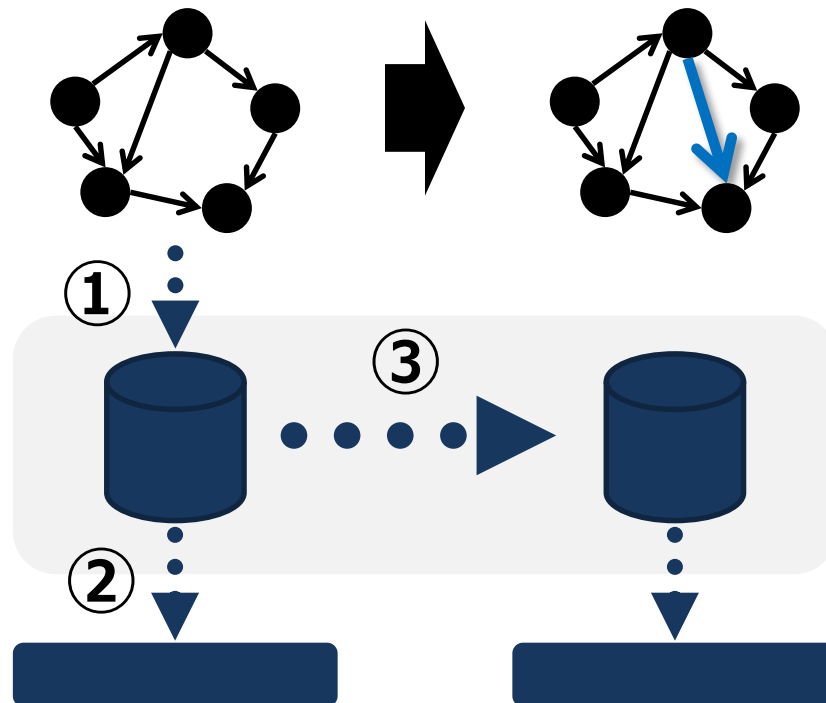
$$\sigma(X + v) - \sigma(X) \geq \sigma(Y + v) - \sigma(Y)$$

Key: fast & accurate estimation of $\sigma(\cdot)$

Proposed method

What we need:

- ① Indexing algorithm
- ② Influence query algorithms
- ③ Update algorithms



Index construction

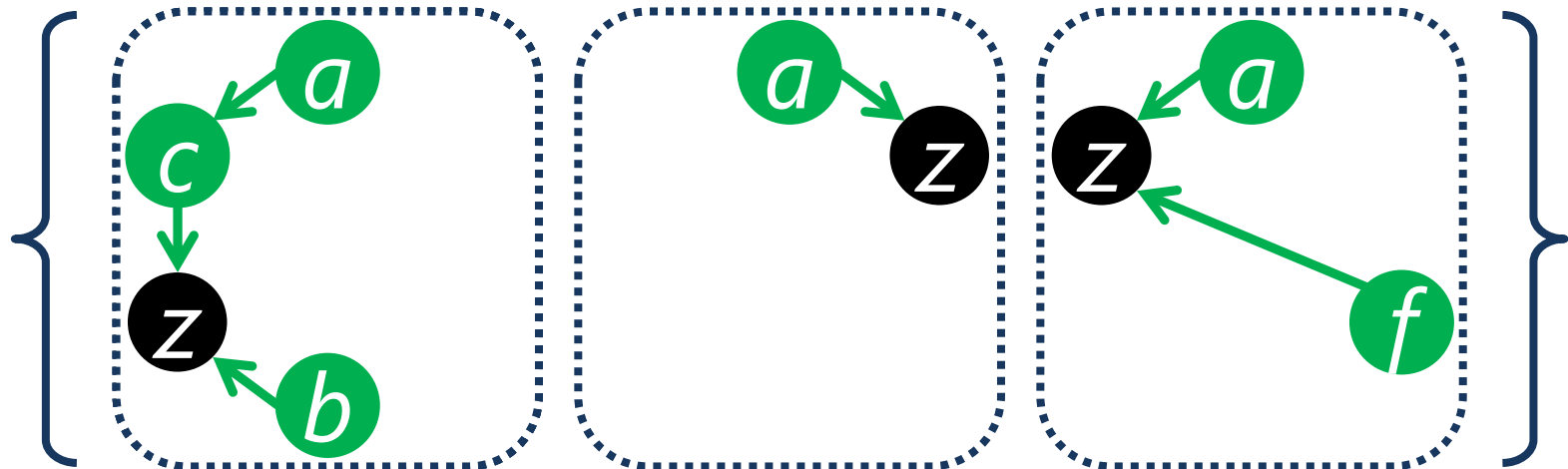
Redesign of *Reverse Influence Sampling*

[Borgs-Brautbar-Chayes-Lucier. *SODA'14*]

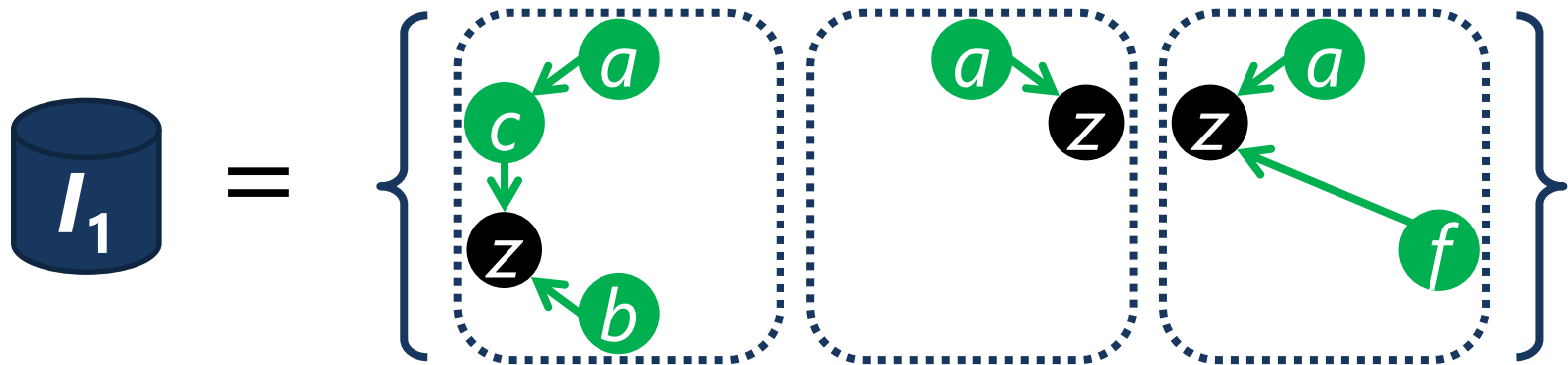
Single sketch construction :

- ▶ Randomly select a **target** vertex z
- ▶ Conduct a **reverse simulation** from z
- ▶ **Sketch** = (● 's) \cup (↗ 's)

Index size = #● + \sum ●'s in-deg = $\Theta(\epsilon^{-3}(|V| + |E|) \log|V|)$



Property of our index



Vertices frequently appearing in sketches are influential

$$\sigma(S) \propto \mathbf{E}[\# \text{ sketches intersecting } S]$$

[Borgs-Brautbar-Chayes-Lucier. *SODA'14*]

Query algorithms for influence analysis

Based on *Reverse Influence Sampling*

[Borgs-Brautbar-Chayes-Lucier. *SODA'14*]

Influence estimation query (seed set S)

- ▶ Computing the size of union on sketches
- ▶ $\sigma(S) \pm \epsilon|V|$ *w.h.p.* (Thm. 5.9)

Influence maximization query (solution size k)

- ▶ Solving maximum coverage on sketches
- ▶ $(1 - e^{-1} - \epsilon)$ -**approx.** *w.h.p.* (Thm. 5.10)

We further introduce speed-up techniques (see our paper)

Overview of index update algorithms

3 edge operations

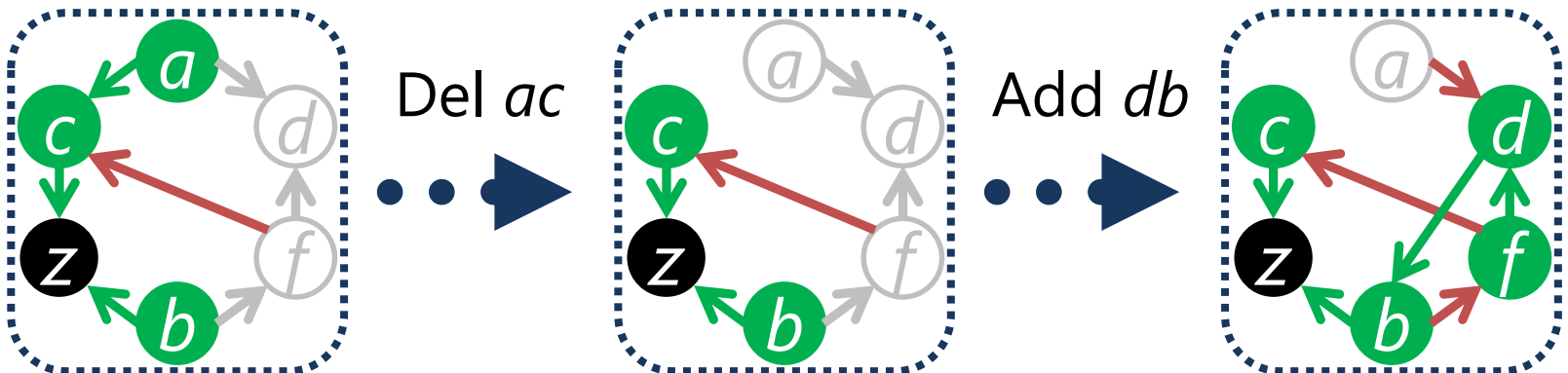
- ▶ Edge addition
- ▶ Edge deletion
- ▶ Probability change

2 vertex operations

- ▶ Vertex addition
- ▶ Vertex deletion

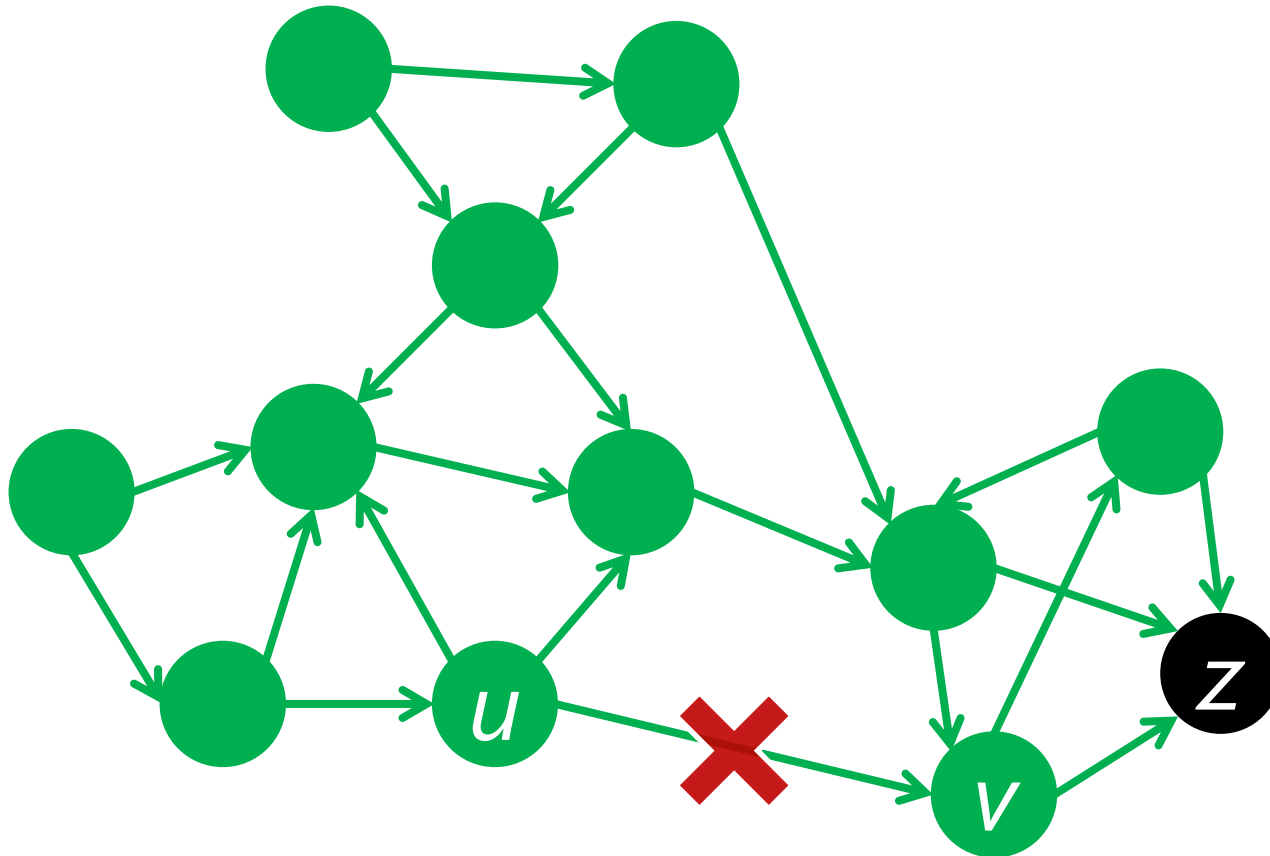
Independently update *each* sketch so that
“all ● can reach ● **z** by passing through ”

↪ Non-degeneracy = **No need reindexing** (Thm. 5.8)



Edge deletion example (1)

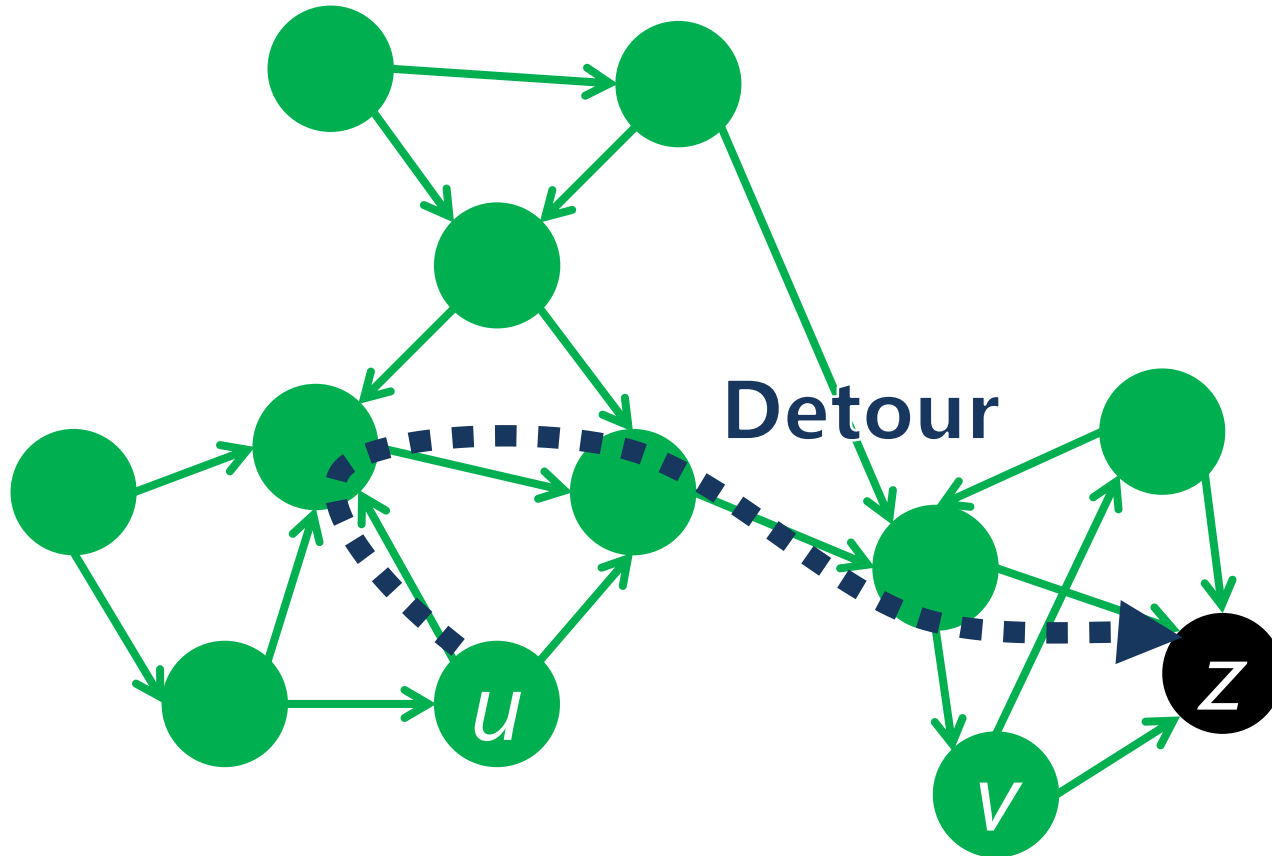
Q. "Vertices that can reach **z**" **decrease**?



Edge deletion example (1)

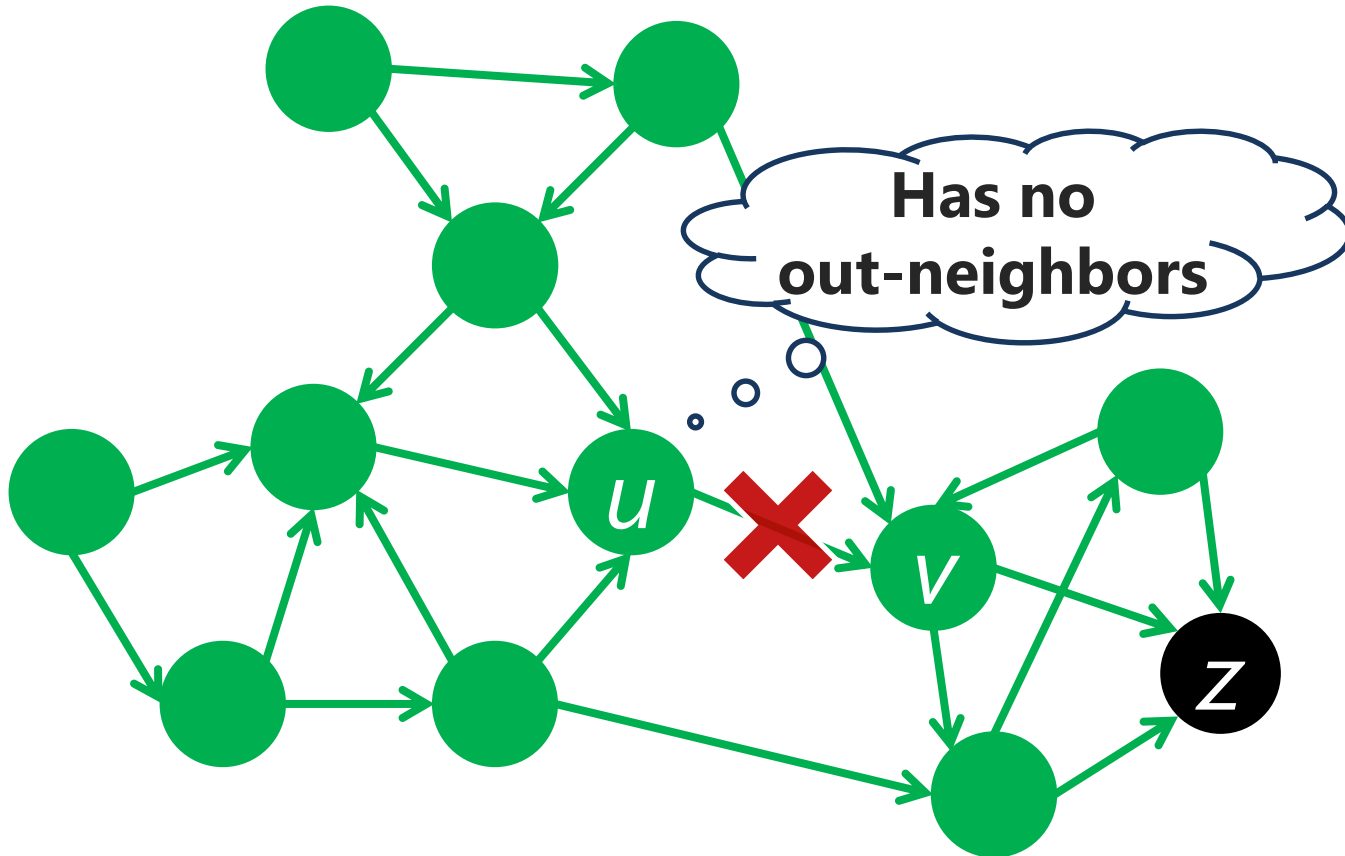
Q. "Vertices that can reach **z**" **decrease**?

A. NO



Edge deletion example (2)

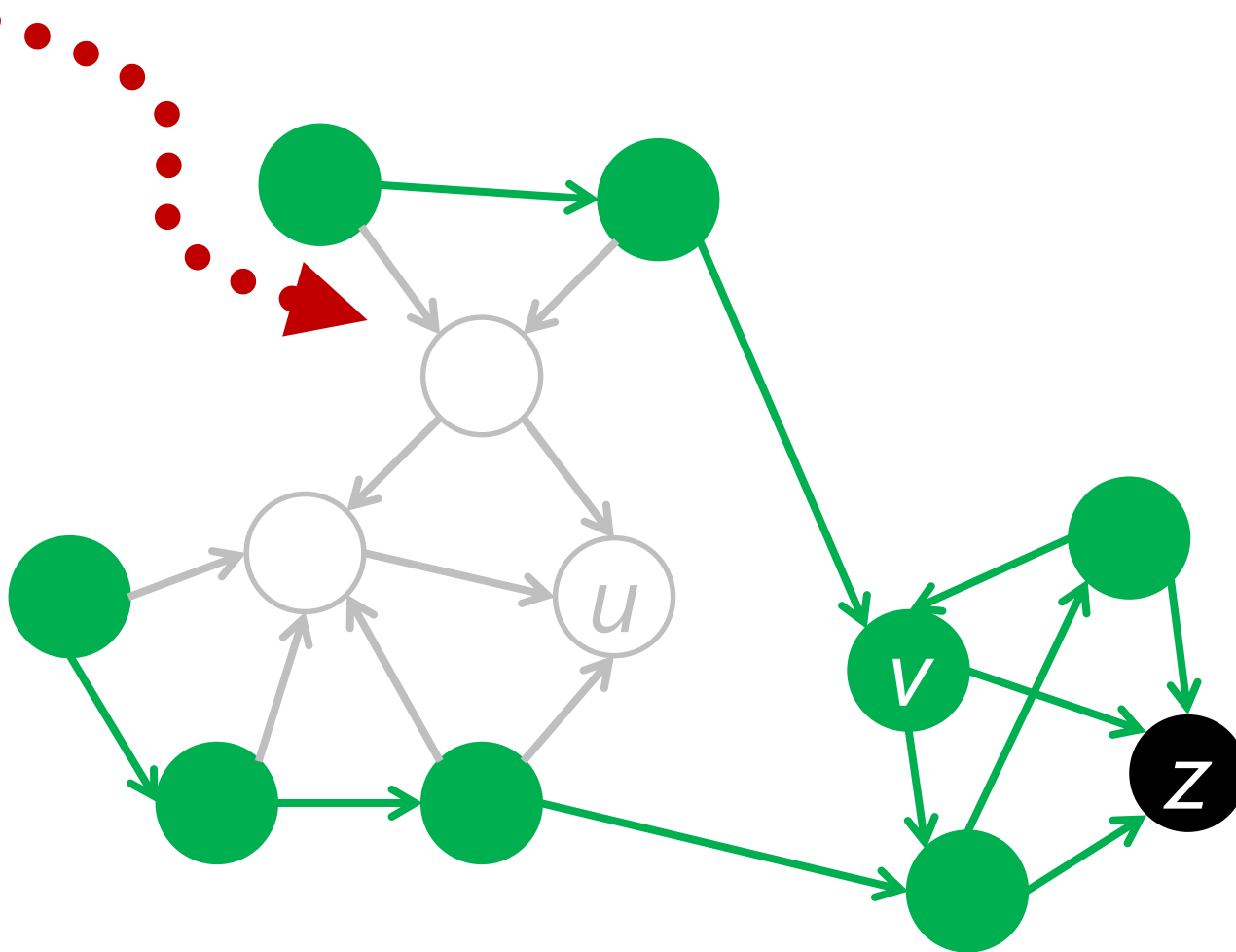
Q. "Vertices that can reach **z**" **decrease**?



Edge deletion example (2)

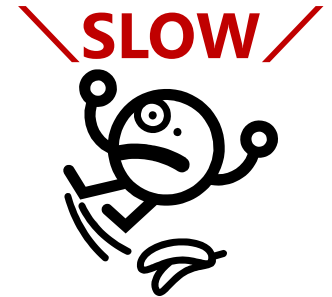
Q. "Vertices that can reach **z**" **decrease**?

A. YES

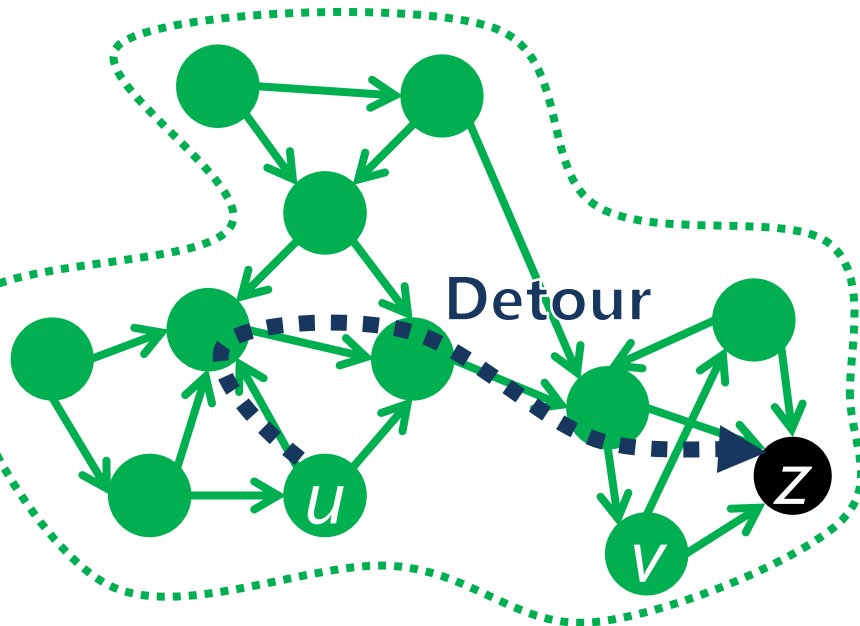


How about naive update for edge deletion?

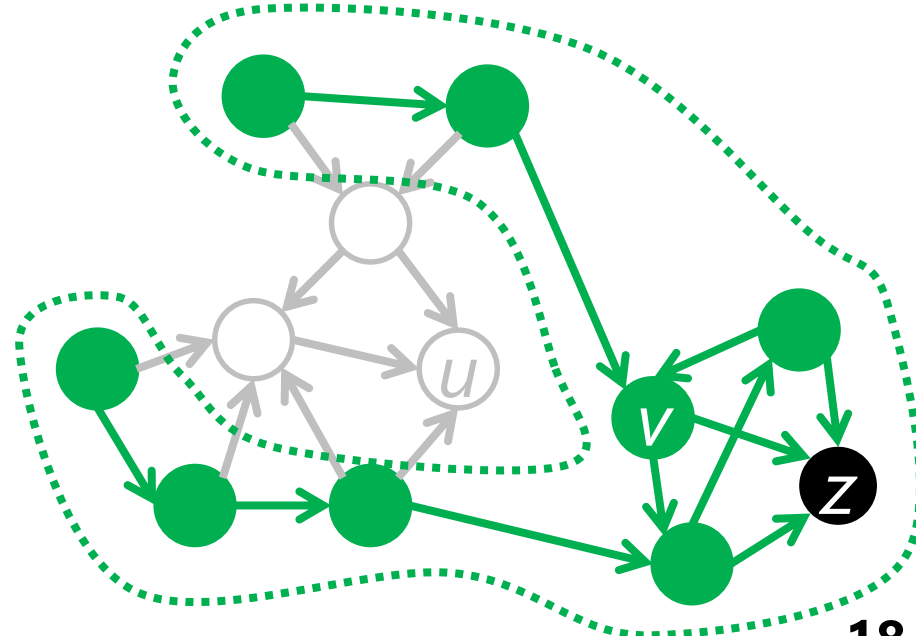
Perform a reverse BFS from **Z**
To detect , need to scan **all** 



Challenge 1:
Detour detection

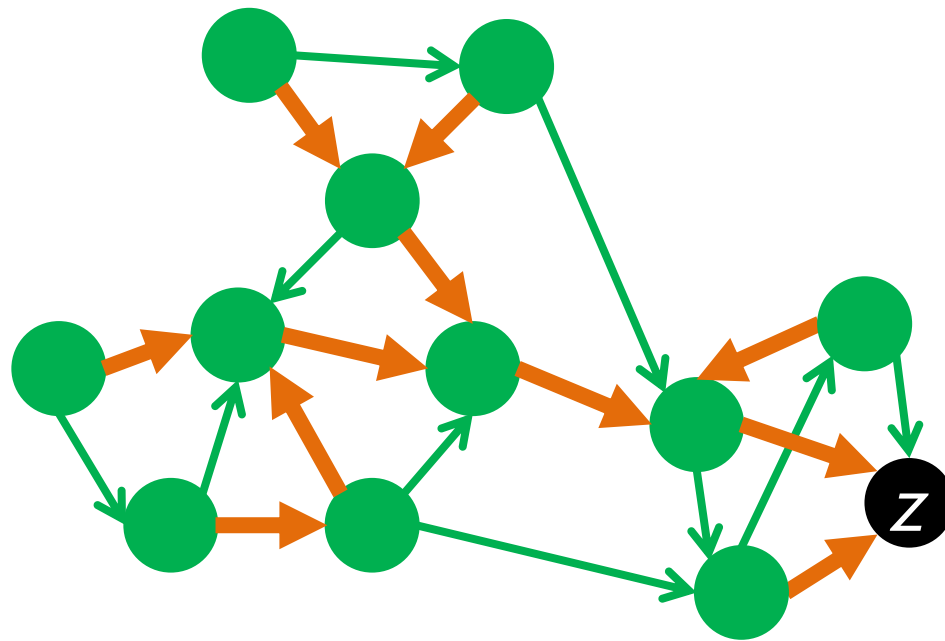


Challenge 2:
Efficient reverse BFSes



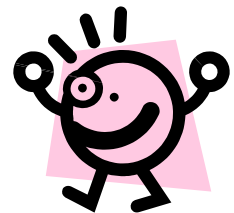
Fast edge deletion: Using **reachability tree**

A **directed subtree** of a sketch rooted on **Z**



- ☺ **Detour existence check**
- ☺ **Limiting the search range**

* Can be used for vertex deletion



Proposed method ③Update algorithms

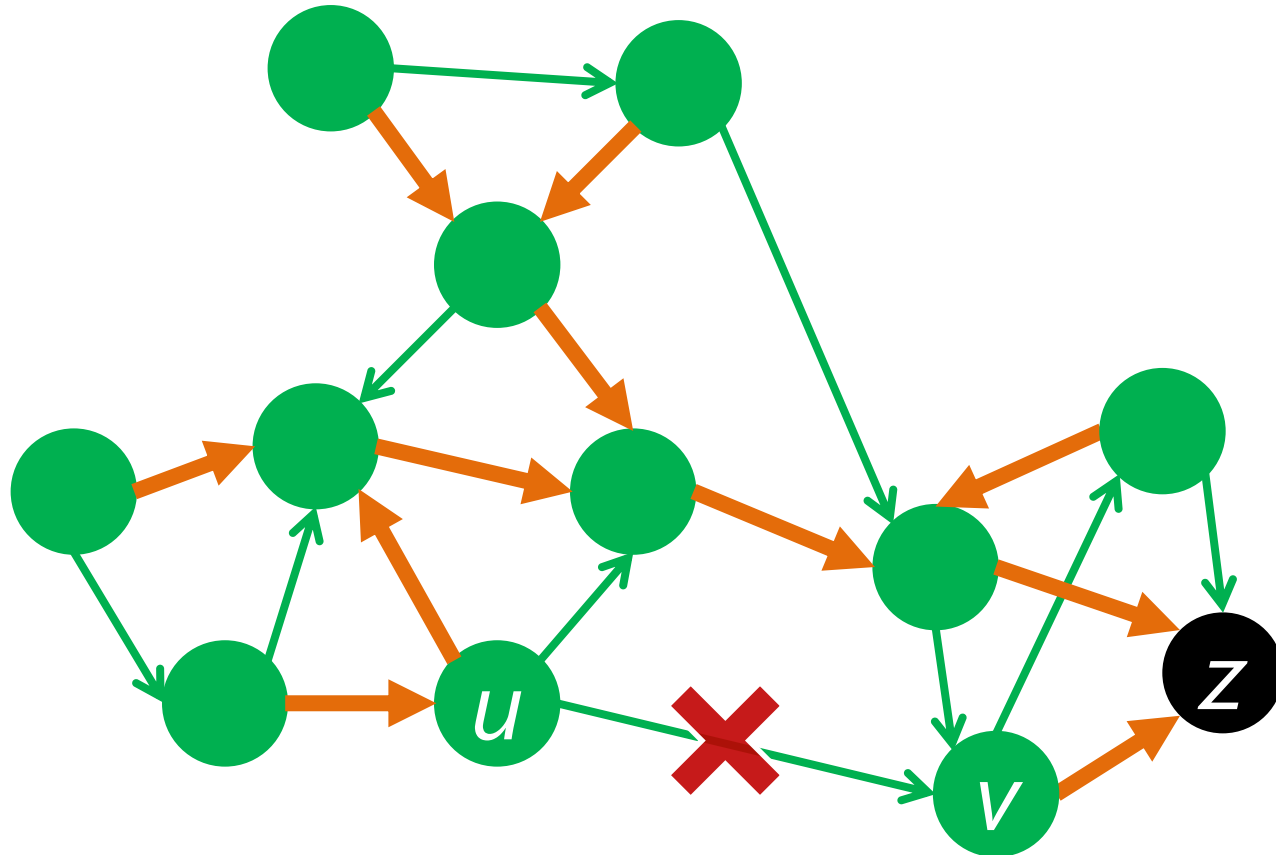
Fast edge deletion:

Detour existence check

$uv \notin \text{tree} \Rightarrow \exists$ a detour from u to z

* not vice versa

10% deletions are pruned



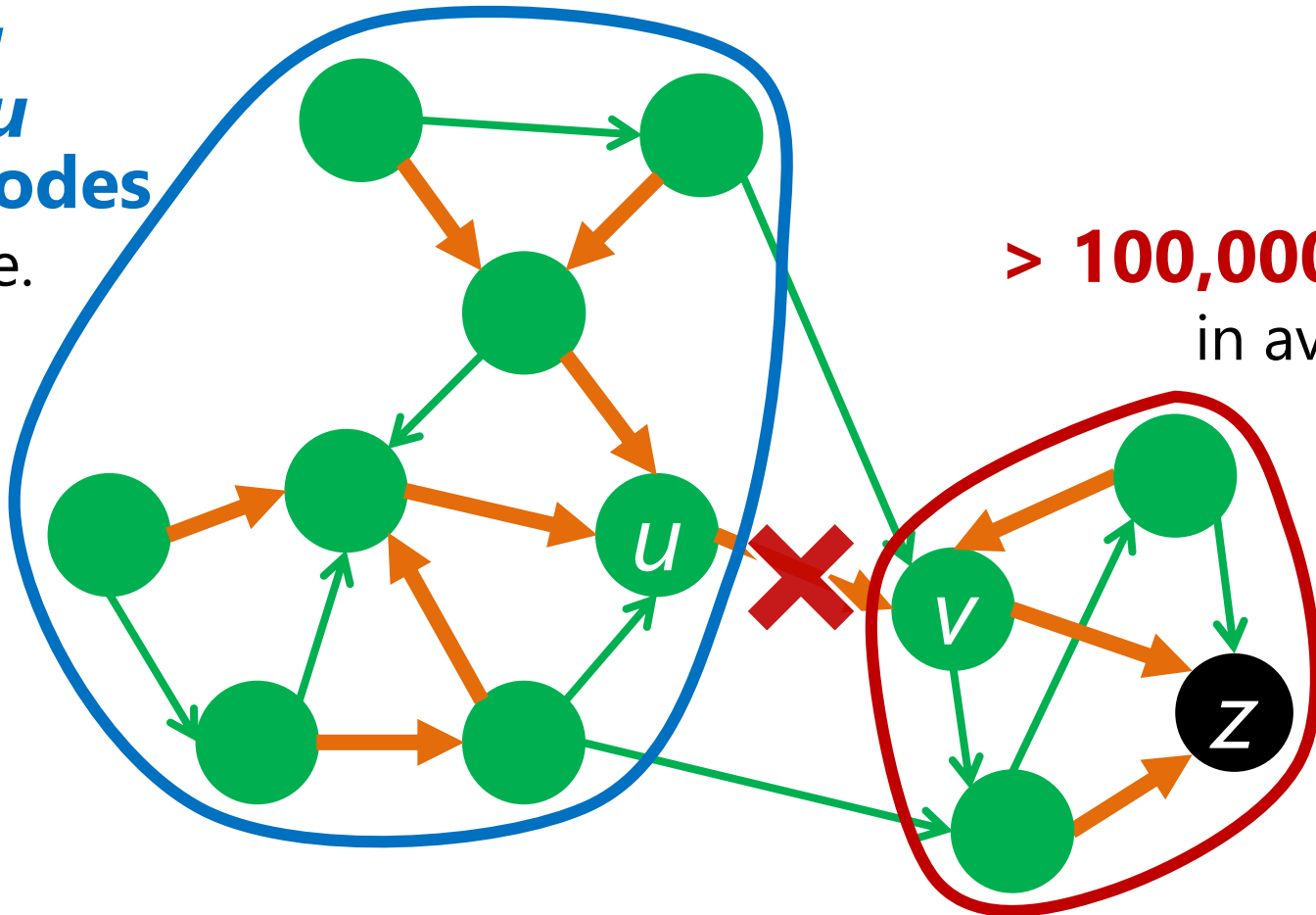
Proposed method ③Update algorithms

Fast edge deletion:

Limiting the search range

Verify a subtree T_u rooted on u & update **tree**

T_u
a few nodes
in ave.



> 100,000 nodes
in ave.

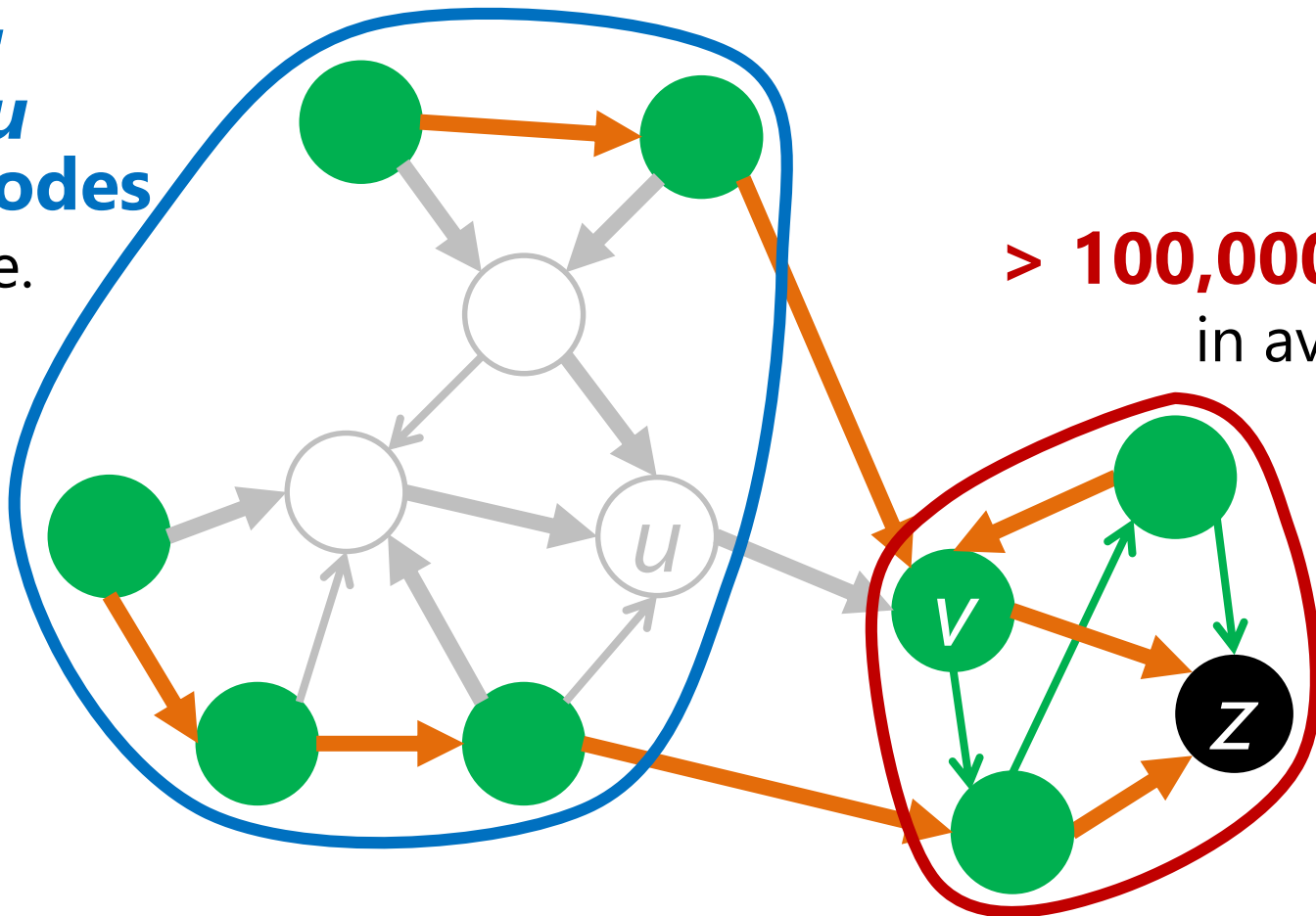
Proposed method ③Update algorithms

Fast edge deletion:

Limiting the search range

Verify a subtree T_u rooted on u & update **tree**

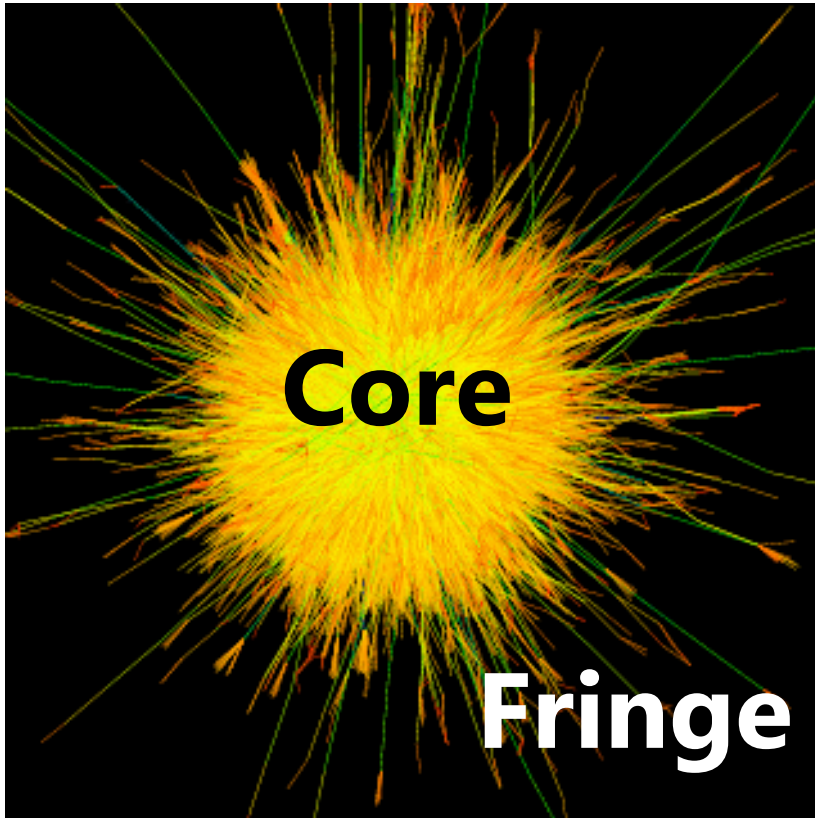
T_u
a few nodes
in ave.



Why our techniques are effective?

Core-fringe structure

[Leskovec-Lang-Dasgupta-Mahoney. *WWW'08*]
[Maehara-Akiba-Iwata-Kawarabayashi. *PVLDB'14*]



Core is **dense**

Many detours

1st tech. works well

Fringe is **tree-like**

T_u is small

2nd tech. works well

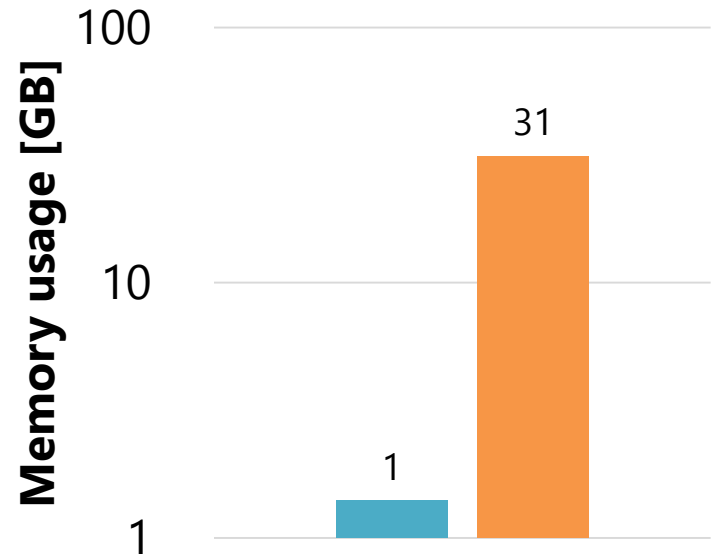
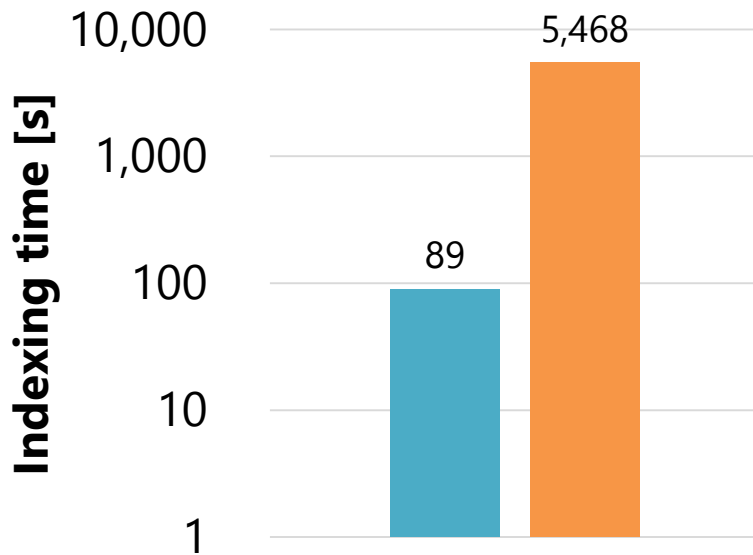
Experiments

Efficiency of {
Index construction
Index update
Influence estimation query
Influence maximization query

- ▶ Dataset : Koblenz Network Collection <http://konect.uni-koblenz.de/>
with timestamps at which edge was created
- ▶ Machine: Intel Xeon E5-2690 2.90GHz CPU + 256GB RAM
- ▶ Compiler: g++v4.6.3 (-O2)
- ▶ Index size = $32(|V| + |E|) \log|V|$
- ▶ Edge prob. = randomly chosen from $\{0.1, 0.01, 0.001\}$

Index construction

Network	$ V $	$ E $
Epinions	130K	840K
Flickr	2,303K	33,140K





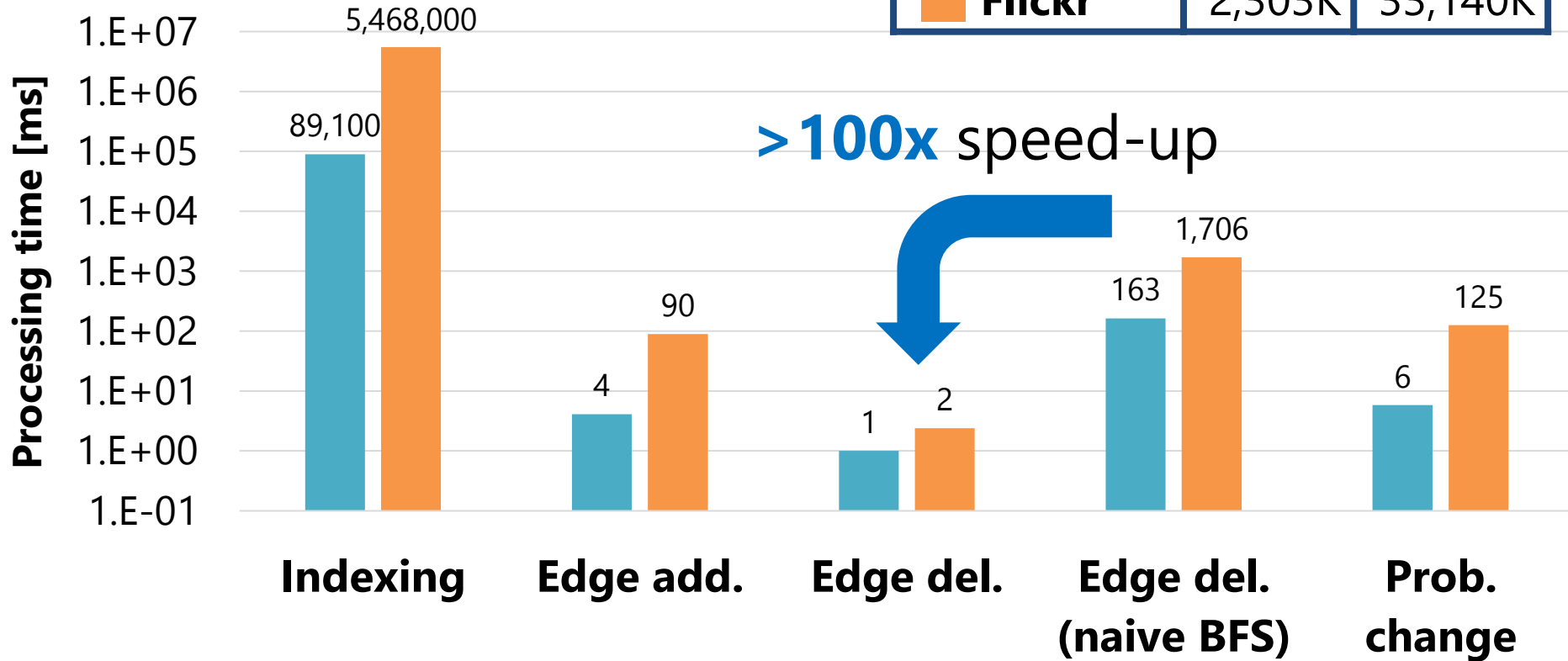
- ▶ Can handle graphs with **tens of millions** of edges
 - ▶ Indexing is required **just once**

Experiments

Index update time:

Edge operations

Network	$ V $	$ E $
 Epinions	130K	840K
 Flickr	2,303K	33,140K



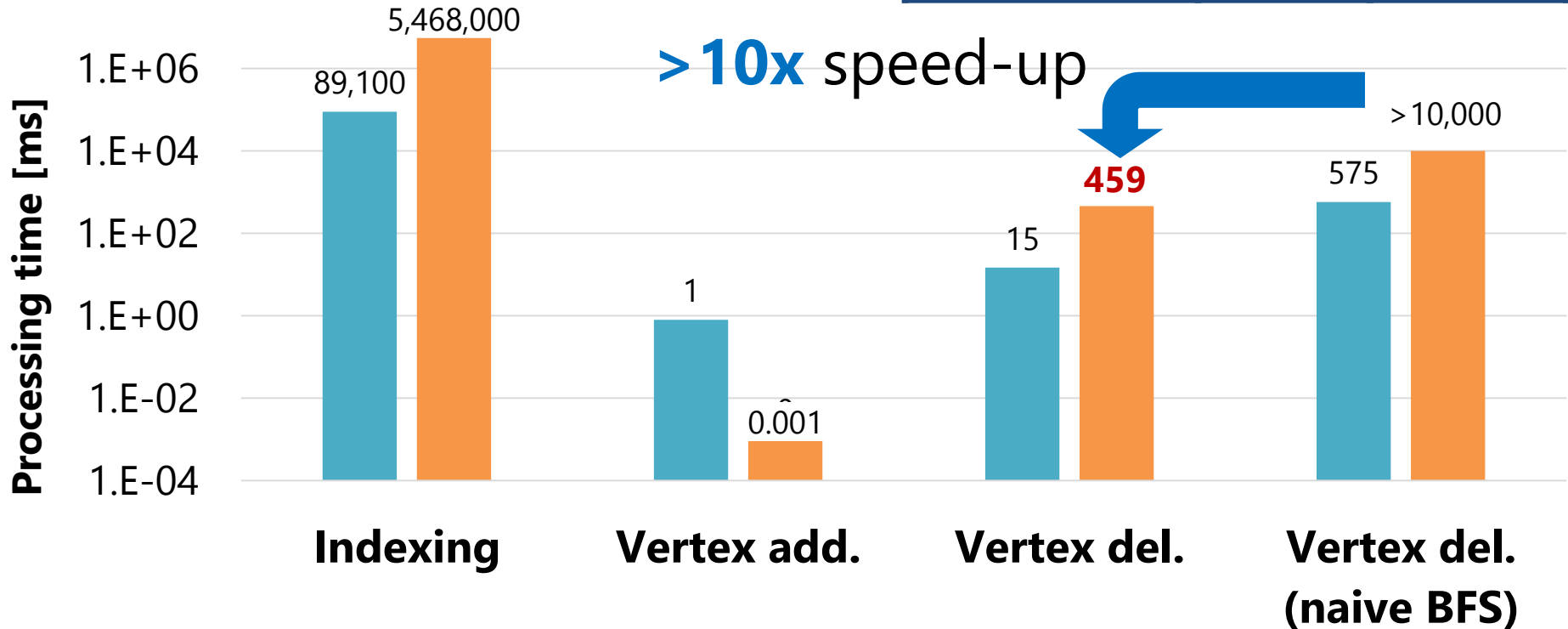
► (Update time) \ll (Indexing time)

Experiments

Index update time:

Vertex operations

Network	$ V $	$ E $
Epinions	130K	840K
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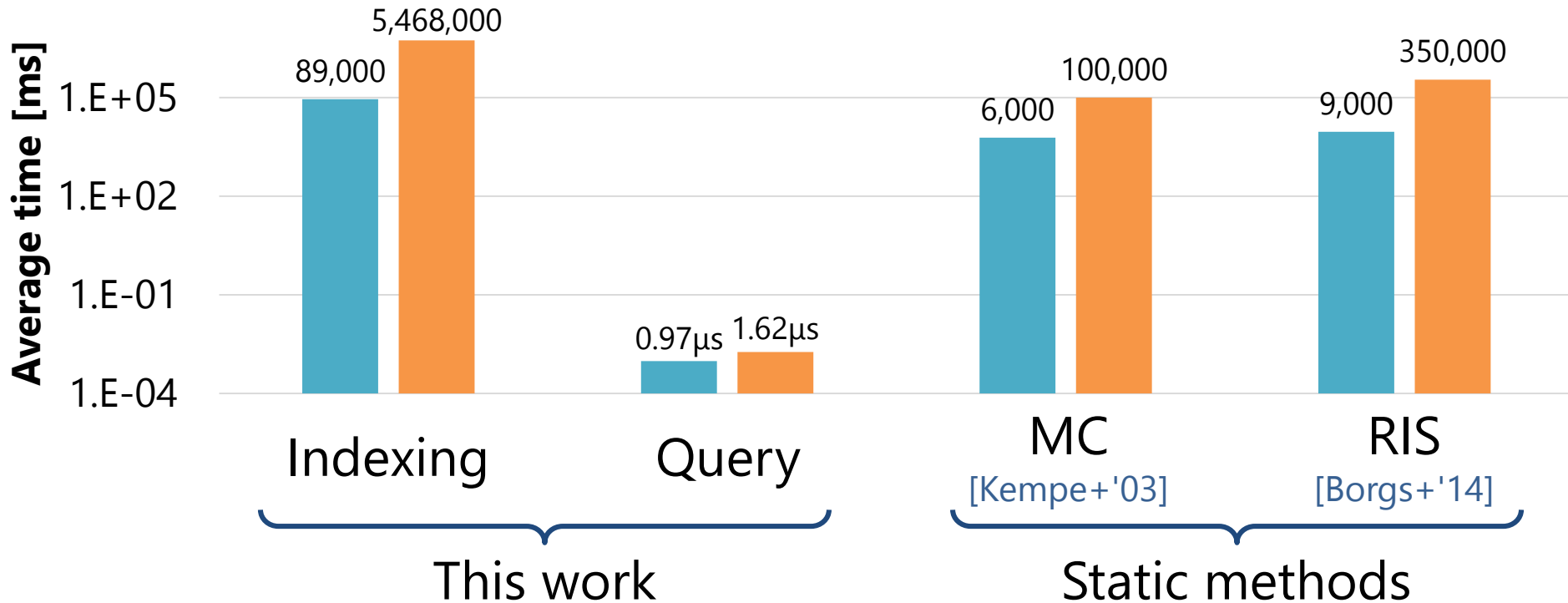
► (Update time) \ll (Indexing time)

► **Vertex del.** causes **a number of edge dels.**

Experiments

Influence estimation queries:

Time for estimating the influence of a vertex



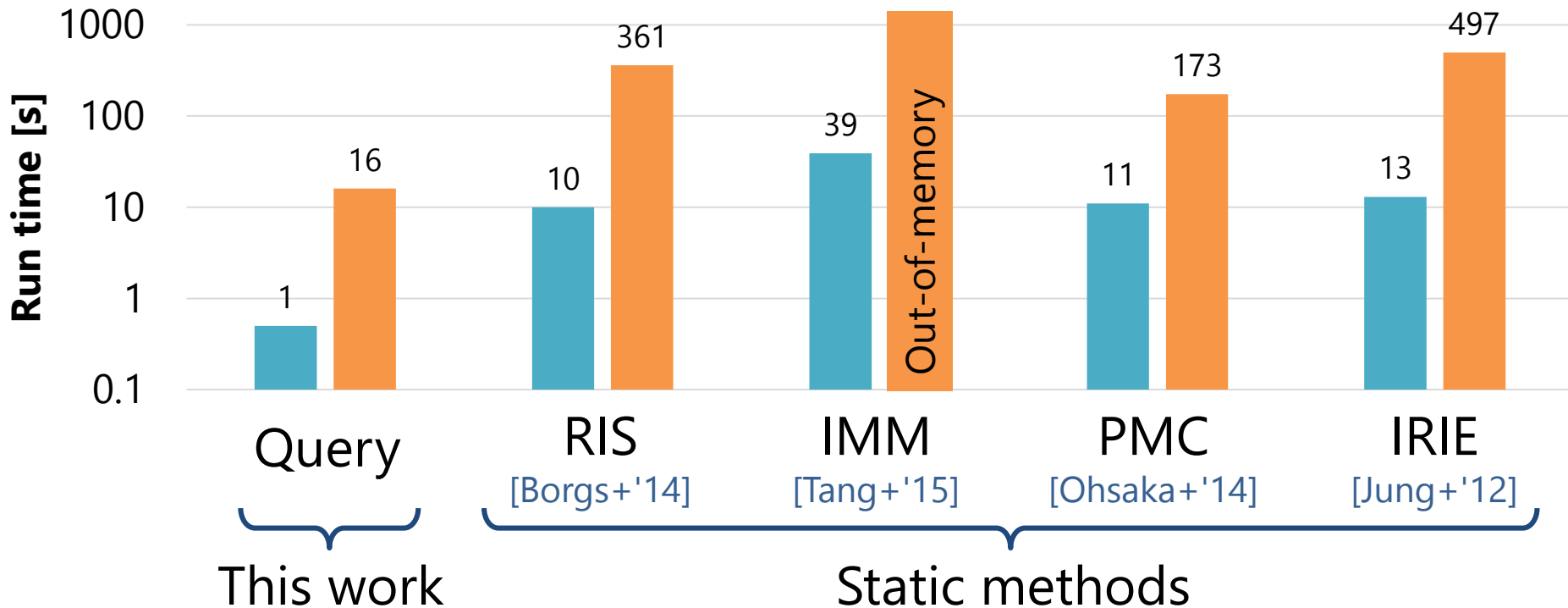
► Tracking **1M nodes/sec**
Just perform **table lookup**

Network	$ V $	$ E $
Epinions	130K	840K
Flickr	2,303K	33,140K

Experiments

Influence maximization queries:

Time for selecting a seed set of size 100



► **> 10 times faster**
Solve max. coverage

Network	$ V $	$ E $
■ Epinions	130K	840K
■ Flickr	2,303K	33,140K

Conclusion

Proposed fully-dynamic indices

for influence analysis in evolving networks

- ① Indexing graphs w/ $> 10M$ edges in a few hours
- ② Reflect any graph change in 1 sec.
- ③ Fast influence analysis queries

Future directions

- ▶ More space saving for billion-scale graphs
- ▶ Fast influence maximization query
Maximum Coverage in online setting